

Quasinormal modes of black holes immersed in a strong magnetic field

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We found quasinormal modes, both in time and frequency domains, of the Ernst black holes, that is neutral black holes immersed in an external magnetic field. The Ernst solution reduces to the Schwarzschild solution, when the magnetic field vanishes. It is found that the quasinormal spectrum for massless scalar field in the vicinity of the magnetized black holes acquires an effective "mass" $\mu = 2Bm$, where m is the azimuthal number and B is parameter describing the magnetic field. We shall show that in the presence of a magnetic field quasinormal modes are longer lived and have larger oscillation frequencies. The perturbations of higher dimensional magnetized black holes by Ortogio and of magnetized dilaton black holes by Radu are considered.

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It is well known that supermassive black holes in centres of galaxies are immersed in a strong magnetic field. Interaction of a black hole and a magnetic field can happen in a lot of physical situations: when an accretion disk or other matter distribution around black hole is charged, when taking into consideration galactic and intergalactic magnetic fields, and, possibly, if mini-black holes are created in particle collisions in the brane-world scenarios. Let us note that in Large Hadron Colliders in the region of particle collisions, the huge accelerating magnetic field is assumed to be screened, yet this does not exclude possibility of interaction of strong magnetic fields and mini-black holes in a great variety of high energy processes, when quantum gravity states are excited.

In addition to highly motivated astrophysical interest to magnetic fields around black holes [1], these fields are important also as a background field testing the black hole geometry. Thus, when perturbed, the magnetic field undergoes characteristic damped oscillations, quasinormal modes, which could be observed in experiments. Quasinormal modes of black holes has gained considerable attention recent few years also because of their applications in string theory through the AdS/CFT correspondence. As nowadays there are a great number of papers on this subject, we refer the reader to the reviews [2] and a few papers [3] where a lot of references to the recent research of quasinormal modes can be found.

In the present paper we consider the massless scalar field perturbations around the Ernst black hole [4], a black hole immersed in an external magnetic field, and around its higher dimensional [7] and dilatonic [8] generalizations. The Ernst metric is the exact solution of the Einstein-Maxwell equations. Now, the properties of the Ernst metric are well studied, since the discovery of the Ernst solution [5], and, different generalizations of the Ernst solution are obtained [6]. In the present paper, we shall find the quasinormal modes of the Ernst black holes

both in time and frequency domain and shall show how magnetic field affect the quasinormal spectrum.

The Ernst metric in four dimensions has the form [4],

$$ds^2 = \Lambda^2 \left(\left(1 - \frac{2M}{r} \right) dt^2 - \left(1 - \frac{2M}{r} \right)^{-1} dr^2 - r^2 d\theta^2 \right),$$

$$- \frac{r^2 \sin^2 \theta}{\Lambda^2} d\phi^2, \quad (1)$$

where the external magnetic field is determined by the parameter B

$$\Lambda = 1 + B^2 r^2 \sin^2 \theta. \quad (2)$$

The vector potential for the magnetic field is given by the formula:

$$A_\mu dx^\mu = - \frac{Br^2 \sin^2 \theta}{\Lambda} d\phi. \quad (3)$$

As a magnetic field is assumed to exist everywhere in space, the above metric is not asymptotically flat. The event horizon is again $r_h = 2M$, and the surface gravity at the event horizon is the same as that for a Schwarzschild metric, namely

$$\chi = 2\pi T_H = \frac{1}{4M}. \quad (4)$$

This leads to the same classical thermodynamic properties [5] as for the case of Schwarzschild black hole. The massless scalar field equation has the form:

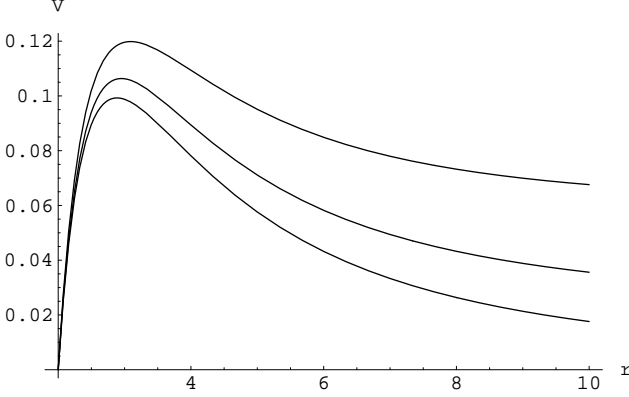
$$\square \Phi \equiv \frac{1}{\sqrt{-g}} (g^{\mu\nu} \sqrt{-g} \Phi_{,\mu})_{,\nu} = 0 \quad (5)$$

In its general form, the Klein-Gordon equation does not allow separation of radial and angular variables for the Ernst background metric. Yet, even very strong magnetic fields in centres of galaxies or in colliders, corresponds to $B \ll M$ in our units, so that one can safely neglect terms higher than B^2 in (5). Indeed, in

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Figure 1: Effective potential for different values of the magnetic field: $B = 0$ (bottom), $B = 0.075$, $B = 0.125$ (top)



the expansion of Λ^4 in powers of B , the next term after that proportional to $B^2 r^2$, is $\sim B^4 r^4$ and, thereby, is very small in the region near the black hole ($\Lambda^4 \approx 1 + 4B^2 m^2 r^2 + O(B^4)$). The term $\sim B^4 r^4$ is growing far from black hole, and, moreover the potential in the asymptotically far region is diverging, what creates a kind of confining by the magnetic field of the Ernst solution. This happens because the non-decaying magnetic field is assumed to exist everywhere in the Universe. Therefore it is clear that in order to estimate a real astrophysical situation, one needs to match the Ernst solution with a Schwarzschild solution at some large r . Fortunately we do not need to do this for the quasinormal mode problem: the quasinormal spectrum of astrophysical interest is stipulated by the behavior of the effective potential in some region *near black hole*, while its behavior far from black hole is insignificant [9]. In this way we take into consideration only dominant correction due to magnetic field to the effective potential of the Schwarzschild black hole. The exact Klein-Gordon equation for the angular part reads,

$$\frac{P_{sch}(\theta, \phi)\Phi}{r^2} + \frac{\Lambda^4 - 1}{r^2 \sin^2 \theta} \partial_{\phi\phi} \Phi = 0, \quad (6)$$

with $P_{sch}(\theta, \phi)$ meaning the corresponding pure-Schwarzschild part of the angular equation. Thus, $P_{sch}(\theta, \phi)\Phi = -l(l+1)\Phi$ and neglecting in (6) terms $\sim B^4$ and higher, after separation of the angular variables, we reduce the wave equation (5) to the Schroedinger wave equation

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V(r^*) \right) \Psi(r^*) = 0, \quad (7)$$

with the effective potential V :

$$V(r) = f(r) \left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + 4B^2 m^2 \right), \quad (8)$$

where

$$f(r) = 1 - \frac{2M}{r}, \quad dr_* = \frac{dr}{f(r)}$$

and m is the azimuthal quantum number coming from the Killing vector ∂_ϕ of the Ernst metric. We can see that the effective potential (8) coincides with the potential for the *massive scalar field with the effective mass $\mu = 2Bm$ in the Schwarzschild background*. Let us note also, that neglecting higher terms in powers of B^2 we obtained the astro-physically relevant wave equation which satisfies the quasinormal mode boundary conditions: purely outgoing wave at spatial infinity and pure in-going waves at the event horizon. The wave equation for the exact Ernst metric would have diverging effective potential (this can be seen for example from the full effective potential of the particle moving in the equatorial plane around the Ernst black hole [10]) and would require the Dirichlet confining boundary conditions.

In exactly the same way one can study the propagation of a scalar field in the dilatonic Ernst background [8]. The metric has the form,

$$ds^2 = \Lambda^{\frac{2}{1+a^2}} \left(\left(1 - \frac{2M}{r} \right) dt^2 - r^2 d\theta^2 - \left(1 - \frac{2M}{r} \right)^{-1} dr^2 - \frac{r^2 \sin^2 \theta}{\Lambda^{\frac{2}{1+a^2}}} d\phi^2 \right), \quad (9)$$

where

$$\Lambda = 1 + (1 + a^2) B^2 r^2 \sin^2 \theta, \quad (10)$$

and a is a constant factor which relates the dilaton and the magnetic field as $\frac{\phi}{\ln \Lambda} = -\frac{a}{1+a^2}$. The resulting effective potential is exactly the same as potential (8), so that the dilaton parameter a cannot be seen at the dominant order in B^2 expansion.

The same occurs for the D -dimensional Ernst metric found by Ortoggio [7],

$$ds^2 = \Lambda^{\frac{2}{D-3}} [-F(r) dt^2 + \{F(r)\}^{-1} dr^2 + r^2 d\theta^2 + r^2 \cos^2 \theta d\Omega_{D-4}^2] + \Lambda^{-2} r^2 \sin^2 \theta d\Phi^2, \quad (11)$$

where $d\Omega_{D-4}^2$ is the line element of the $(D-4)$ -sphere,

$$d\Omega_{D-4}^2 = d\Psi_1^2 + \prod_{a=1}^{D-5} \sin^2 \Psi_a d\Psi_{a+1}^2, \quad (12)$$

the function $F(r)$ is

$$F(r) = 1 - \frac{16\pi M(D-2)^{-1}}{r^{D-3}\Omega_{D-2}}, \quad (13)$$

and Λ is

$$\Lambda = 1 + \frac{4(D-3)}{2D-4} B^2 r^2 \sin^2 \theta. \quad (14)$$

The potential for the scalar field propagation has the form

$$V(r) = F(r) \left(\frac{\ell(\ell+D-3)}{r^2} + \frac{dF(r)}{dr} \frac{D-2}{2r} + 4B^2 m^2 + \frac{F(r)(D-4)(D-2)}{4r^2} \right). \quad (15)$$

Table I: Quasinormal modes for Ernst black holes for different values of the magnetic field B , $M = 1$, $D = 4$.

B	$\ell = 1, m = 1$	$\ell = 2, m = 1$
0.005	0.292981 - 0.097633 i	0.484433 - 0.096488 i
0.025	0.294054 - 0.096988 i	0.486804 - 0.095675 i
0.050	0.297416 - 0.094957 i	0.490764 - 0.094312 i
0.075	0.303040 - 0.091521 i	0.496327 - 0.092389 i
0.100	0.321199 - 0.080040 i	0.496327 - 0.092389 i
0.125	0.333777 - 0.071658 i	0.503512 - 0.089891 i

If we again neglect the terms $\sim B^4$ and higher, the quasinormal modes also have the same form for D-dimensional, Dilatonic or “pure Ernst” geometry. Now we are in position to use all the available data for the massive scalar quasinormal modes in the Schwarzschild black hole. The quasinormal modes for massive scalar fields were studied for the first time by Will and Simone [11] and later in [12] with the help of the WKB method [13]. The massive quasinormal modes are characterized by the growing the damping time with the mass until the appearance of the infinitely long lived modes called quasi-resonances [14]. In [15] it was shown that when increasing the field mass, the damped quasinormal modes disappear “one by one” and a single corresponding quasi-resonance appears instead, leaving all the remaining higher overtones damped.

Note, that the wave equation with the obtained potential (8) satisfies the quasinormal mode boundary condition at spatial infinity, which in our particular case takes the form,

$$\Psi(r^*) \sim C_+ e^{i\chi r^*} r^{(iMm^2/\chi)}, \quad (r, r^* \rightarrow +\infty) \quad (16)$$

$$\chi = \sqrt{\omega^2 - m^2}. \quad (17)$$

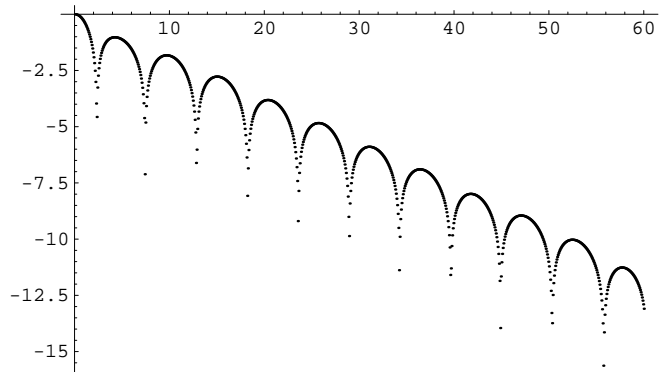
(The sign of χ is to be chosen to stay in the same complex surface quadrant as ω .) Within the Frobenius method we expand the wave function as follows [15],

$$\Psi(r) = e^{i\chi r} r^{(2iM\chi + iMm^2/\chi)} \times \left(1 - \frac{2M}{r}\right)^{-2iM\omega} \sum_n a_n \left(1 - \frac{2M}{r}\right)^n, \quad (18)$$

We obtained the quasinormal modes for different values of B , ℓ and m . The results are summarized in the following table

The alternative time-domain description is based on the numerical integration scheme described, for instance in [16]. The obtained results show very good agreement with the accurate numerical values of the table. For instance, for $B = 0.05$ $m = \ell = 1$, we have $0.295 - 0.096i$ with the time domain integration and $0.297416 - 0.094957i$ with the Frobenius method. The 6th order WKB formula for these values of parameters

Figure 2: Evolution of perturbations for $B = 0.05$, $M = 1$, $m = 1$, $\ell = 1$, $D = 4$.



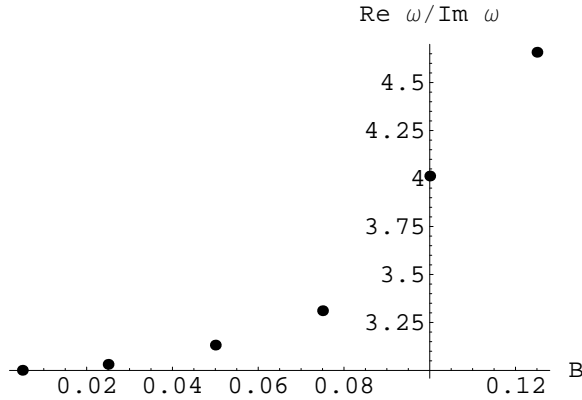
gives $0.2974 - 0.0951i$, what is very close to the accurate Frobenius value. (The third order WKB formula certainly gives less accurate value $0.2956 - 0.09527i$). Let us note, that the WKB method is expected to be much less accurate when applying to the massive scalar field, because it implies pure exponential asymptotics at both infinities and does not take into consideration the subdominant asymptotic included in the pre-factor in 16. Nevertheless, when we are limited by relatively small values of B , the 6th WKB formula is still a very good approximation.

The obtained numerical values of Table I is accurate as they are based on the converging series expansion. The time domain integration usually is less accurate.

From the available data we can see that the $Re\omega$ grows and $Im\omega$ decreases when increasing the magnetic field B . Therefore a black hole is a better oscillator in the presence of a magnetic field, i.e. in this case the quality factor $Q \sim Re\omega/Im\omega$ is considerably increased (see Fig. 2). Let us note, that despite the azimuthal quantum number m can be large, it requires $|m| \leq \ell$, i.e. large m will require large ℓ , so that the term $\sim 4B^2m^2$ will always be small in comparison with the centrifuge term $\sim \ell(\ell+1)r^{-2}$. It means that the obtained effect is a small correction to the Schwarzschild geometry.

It is essential that from the effective potential in Fig. 1, one can see that for not very large B the maximum of the potential is slightly displaced from its Schwarzschild position $r = 3M$. As was shown in [9], the quasinormal modes are stipulated by the behavior of the effective potential near its maximum, because the resonances of the scattering processes naturally depend on the behavior of the potential near its pick. A good illustration of this in [9] shows that some potential given numerically or by fit in such a way that it coincides with the Regge-Wheeler potential near the black hole until $r = 5M$ and diverges at large r has the same quasinormal modes as the Schwarzschild black hole. This must persuade us that it is enough to trust the Ernst solution until a few radius far from the black hole and that one can safely neglect

Figure 3: $\text{Re } \omega / \text{Im } \omega$ for different values of the magnetic field B : $M = 1$, $m = 1$, $\ell = 1$, $D = 4$.



behavior in the further region.

We have written down here only QNM values for $m = 1$ and some first ℓ . Modes with higher m can be easily obtained by re-definition of the "mass term" $\mu_{eff} = 2Bm$.

Finally in Figures 3, 4, one can see the time domain profile for the perturbations of higher dimensional generalization of the Ernst black holes. The decay rate is again smaller in the presence of the magnetic field, while the real oscillation frequencies are larger, so that the quality factor is increased when the magnetic field is increased. As there is quite complete numerical data on

quasinormal modes of massive scalar field for higher dimensional Schwarzschild black holes in [17], we showed here only some time-domain profiles which give excellent agreement with the numerical data of [17].

Conclusion

In the present paper we have found the quasinormal modes for the Ernst black hole and its higher dimensional generalization. This describes the influence of the strong magnetic field onto characteristic quasinormal spectrum of black holes. In particular, the real oscillation frequency grows when increasing the magnetic field. The damping rate is decreasing, when the magnetic field is increasing, so that magnetized black hole is characterized by longer lived modes with higher oscillation frequencies, i.e. by larger quality factor.

The obtained wave equation for the scalar field allows to find the absorption cross-sections and quantum corrections to the entropy of black holes due to scalar field in the vicinity of a strong magnetic field [18].

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- [1] A. N. Aliev and D. V. Galtsov, Magnetized Black Holes, *Sov. Phys. Usp.* **32** (1), 75 (1989); A. N. Aliev and N. Ozdemir, *Mon. Not. Roy. Astron. Soc.* **336**:241-248, 2002; R. A. Konoplya, *Phys. Rev. D* **74**, 124015 (2006) [arXiv:gr-qc/0610082];
 - [2] K. D. Kokkotas and B. G. Schmidt, *Living Rev. Relativity* **2**, 2 (1999).
 - [3] E. Abdalla, C. B. M. Chirenti and A. Saa, arXiv:gr-qc/0703071; C. G. Shao, B. Wang, E. Abdalla and R. K. Su, *Phys. Rev. D* **71**, 044003 (2005); S. Musiri, S. Ness and G. Siopsis, *Phys. Rev. D* **73**, 064001 (2006); H. T. Cho, A. S. Cornell, J. Doukas and W. Naylor, arXiv:hep-th/0701193; R. Konoplya, *Phys. Rev. D* **71**, 024038 (2005) [arXiv:hep-th/0410057]; F. Moura and R. Schiappa, *Class. Quant. Grav.* **24**, 361 (2007); S. Das and S. Shankaranarayanan, *Class. Quant. Grav.* **22**, L7 (2005) [arXiv:hep-th/0410209];
 - [4] F. J. Ernst, *Journ. of Math. Phys.* v. 17, p. 54 (1976).
 - [5] D. V. Galtsov, "SU(2,1) Symmetry Of Einstein-Maxwell Equations And Thermodynamics Of Black Holes In A Magnetic Field. (Talk, In Russian);", W. A. Hiscock, *J. Math. Phys.* **22** (1981) 1828. E. Radu, *Mod. Phys. Lett. A* **17** (2002) 2277 [arXiv:gr-qc/0211035];
 - [6] A. K. Gorbatsievich, S. M. T. Ho and E. Schmutzer, *Acta Phys. Polon. B* **26** (1995) 1439.
 - [7] M. Ortaggio, *JHEP* **0505** (2005) 048;
 - [8] M. Agop, E. Radu and R. Slagter, *Mod. Phys. Lett. A* **20** (2005) 1077.
 - [9] R. A. Konoplya and A. Zhidenko, *Phys. Lett. B* **648**, 236 (2007) [arXiv:hep-th/0611226]; R. A. Konoplya and A. Zhidenko, *Phys. Lett. B* **644**, 186 (2007) [arXiv:gr-qc/0605082].
 - [10] D. V. Galtsov, V. I. Petuhov, *Zh.E.T.F.*, p. 801, v. 74 (1978); R. A. Konoplya, *Phys. Lett. B* **644**, 219 (2007) [arXiv:gr-qc/0608066].
 - [11] L. E. Simone and C. M. Will, *Class. Quant. Grav.* **9**, 963 (1992);
 - [12] R. A. Konoplya, *Phys. Lett. B* **550**, 117 (2002) [arXiv:gr-qc/0210105];
 - [13] B.F. Schutz and C.M. Will, *Astrophys. J. Lett.* **291**, L33 (1985); S. Iyer and C.M. Will, *Phys. Rev. D* **35**, 3621 (1987); R. A. Konoplya, *J. Phys. Stud.* **8**, 93 (2004); R. A. Konoplya, *Phys. Rev. D* **68**, 024018 (2003) [arXiv:gr-qc/0303052];
 - [14] A. Ohashi and M. a. Sakagami, *Class. Quant. Grav.* **21**, 3973 (2004).
 - [15] R. A. Konoplya and A. V. Zhidenko, *Phys. Lett. B* **609**, 377 (2005) [arXiv:gr-qc/0411059]; R. A. Konoplya and A. Zhidenko, *Phys. Rev. D* **73**, 124040 (2006) [arXiv:gr-qc/0605013].
 - [16] C. Gundlach, R. H. Price and J. Pullin, *Phys. Rev. D* **49**, 883 (1994); R. A. Konoplya, C. Molina, A. Zhidenko *Phys. Rev. D* **75**, 084004 (2007) [arXiv:gr-qc/0602047];
 - [17] A. Zhidenko, *Phys. Rev. D* **74**, 064017 (2006) [arXiv:gr-qc/0607133].
 - [18] R. Fontana and R. A. Konoplya, in preparation

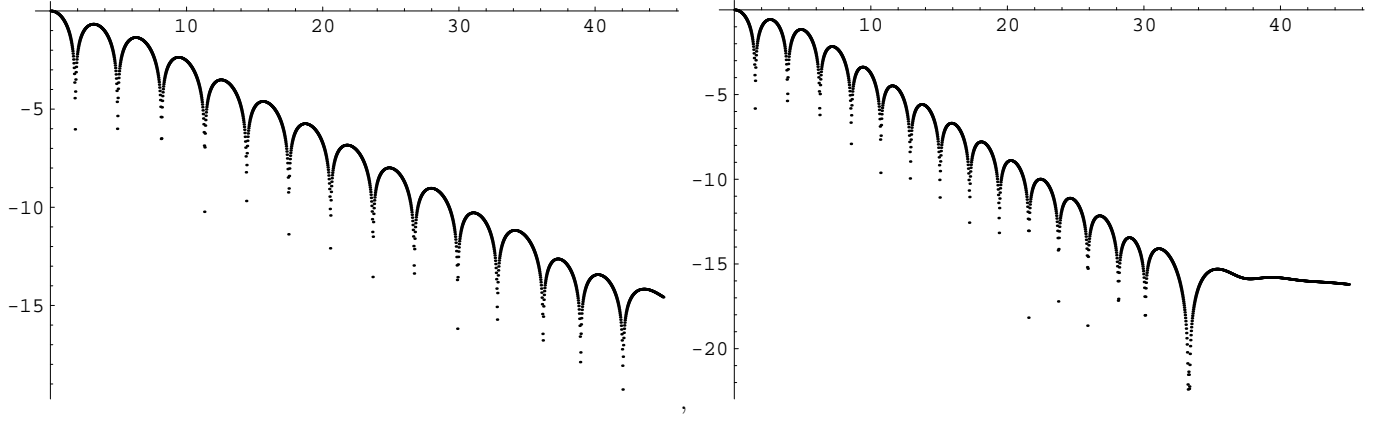


Figure 4: Time-domain profile for quasinormal ringing of the $\ell = m = 1$ modes with $B = 0.05$, $D = 5$ (left), $D = 6$ (right).

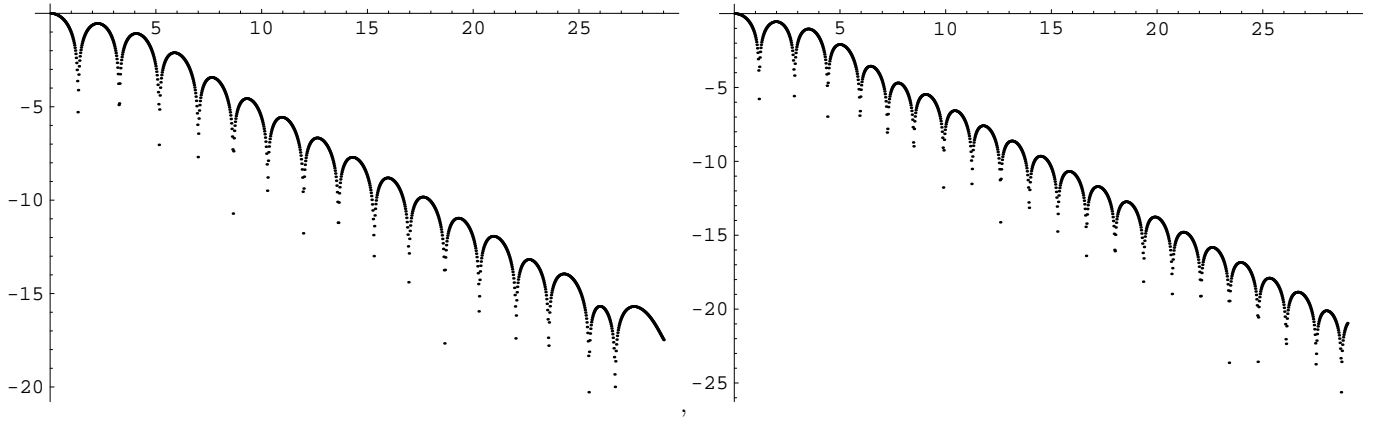


Figure 5: Time-domain profile for quasinormal ringing of the $\ell = m = 1$ modes with $B = 0.05$, $D = 7$ (left), $D = 8$ (right).